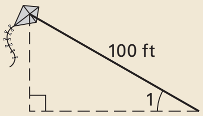
Extra Example 6

A kite is attached to a 100 foot string as shown in the diagram. How far above the ground is the kite when the string forms the given angle?

a. $m\angle 1 = 45°$ **$50\sqrt{2} \approx 70.71$ ft**

b. $m\angle 1 = 30°$ **50 ft**



Animated Geometry classzone.com

An **Animated Geometry** activity is available on-line for **Example 6**. This activity is also available on the **Power Presentations CD-ROM**.

Closing the Lesson

Have students summarize the major points of the lesson and answer the Essential Question: How do you find the lengths of the sides of a 30°-60°-90° triangle and a 45°-45°-90° triangle?

- **In a 45°-45°-90° triangle, the length of the hypotenuse is $\sqrt{2}$ times the length of the leg.**
- **In a 30°-60°-90° triangle, the length of the hypotenuse is 2 times the length of the shorter leg and the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.**

In a 30°-60°-90° triangle the ratio of the sides is $x : x\sqrt{3} : 2x$. In a 45°-45°-90° triangle the ratio of the sides is $x : x : x\sqrt{2}$.

EXAMPLE 6 Find a height

DUMP TRUCK The body of a dump truck is raised to empty a load of sand. How high is the 14 foot body from the frame when it is tipped upward at the given angle?

a. 45° angle **b.** 60° angle

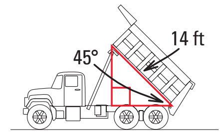
Solution

a. When the body is raised 45° above the frame, the height h is the length of a leg of a 45°-45°-90° triangle. The length of the hypotenuse is 14 feet.

$14 = h \cdot \sqrt{2}$ **45°-45°-90° Triangle Theorem**

$\frac{14}{\sqrt{2}} = h$ **Divide each side by $\sqrt{2}$.**

$9.9 \approx h$ **Use a calculator to approximate.**



REWRITE MEASURES
To write 9.9 ft in feet and inches, multiply the decimal part by 12.
$12 \cdot 0.9 = 10.8$
So, 9.9 ft is about 9 feet 11 inches.

▶ When the angle of elevation is 45°, the body is about 9 feet 11 inches above the frame.

b. When the body is raised 60°, the height h is the length of the longer leg of a 30°-60°-90° triangle. The length of the hypotenuse is 14 feet.

hypotenuse = 2 • shorter leg **30°-60°-90° Triangle Theorem**

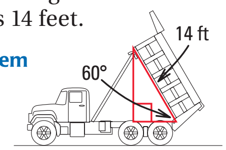
$14 = 2 \cdot s$ **Substitute.**

$7 = s$ **Divide each side by 2.**

longer leg = shorter leg • $\sqrt{3}$ **30°-60°-90° Triangle Theorem**

$h = 7\sqrt{3}$ **Substitute.**

$h \approx 12.1$ **Use a calculator to approximate.**

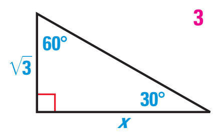


▶ When the angle of elevation is 60°, the body is about 12 feet 1 inch above the frame.

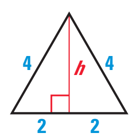
Animated Geometry at classzone.com

✓ GUIDED PRACTICE for Examples 4, 5, and 6

Find the value of the variable.

5. 

3

6. 

$2\sqrt{3}$

7. **WHAT IF?** In Example 6, what is the height of the body of the dump truck if it is raised 30° above the frame? **7 ft**

8. In a 30°-60°-90° triangle, *describe* the location of the shorter side. *Describe* the location of the longer side? ***Sample answer:*** **The shorter side is adjacent to the 60° angle; the longer side is adjacent to the 30° angle.**

7.4 EXERCISES

HOMEWORK KEY
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 5, 9, and 27
★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 6, 19, 22, 29, and 34

SKILL PRACTICE

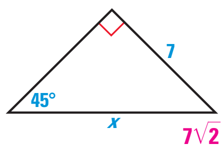
A

1. **VOCABULARY** Copy and complete: A triangle with two congruent sides and a right angle is called _?_. **an isosceles right triangle**

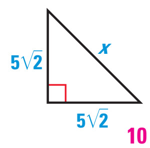
2. ★ **WRITING** *Explain* why the acute angles in an isosceles right triangle always measure 45°. **See margin.**

EXAMPLES 1 and 2 on pp. 457–458 for Exs. 3–5

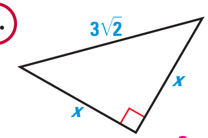
45°-45°-90° TRIANGLES **Find the value of *x*. Write your answer in simplest radical form.**

3. 

$7\sqrt{2}$

4. 

10

5. 

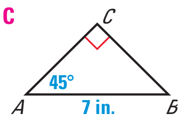
3

EXAMPLE 3 on p. 458 for Exs. 6–7

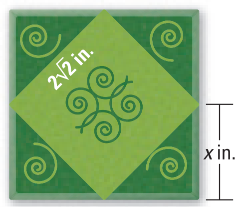
6. ★ **MULTIPLE CHOICE** Find the length of $\overline{AC}$. **C**

(A) $7\sqrt{2}$ in. (B) $2\sqrt{7}$ in.

(C) $\frac{7\sqrt{2}}{2}$ in. (D) $\sqrt{14}$ in.

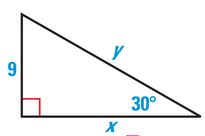


7. **ISOSCELES RIGHT TRIANGLE** The square tile shown has painted corners in the shape of congruent 45°-45°-90° triangles. What is the value of *x*? What is the side length of the tile? **2; 4 in.**

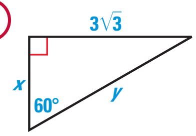


EXAMPLES 4 and 5 on p. 459 for Exs. 8–10

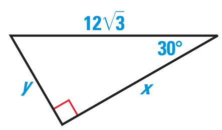
30°-60°-90° TRIANGLES **Find the value of each variable. Write your answers in simplest radical form.**

8. 

$x = 9\sqrt{3}$, $y = 18$

9. 

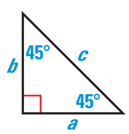
$x = 3$, $y = 6$

10. 

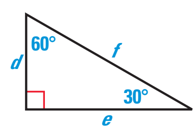
$x = 18$, $y = 6\sqrt{3}$

SPECIAL RIGHT TRIANGLES **Copy and complete the table.** **11, 12. See margin.**

B

11. 

a	7	?	?	?	$\sqrt{5}$
b	?	11	?	?	?
c	?	?	10	$6\sqrt{2}$	?

12. 

d	5	?	?	?	?
e	?	?	$8\sqrt{3}$	?	12
f	?	14	?	$18\sqrt{3}$	?

4 PRACTICE AND APPLY

Assignment Guide

Answer Transparencies available for all exercises

Basic:
Day 1: SRH p. 874 Exs. 17, 18, 22–24
pp. 461–464
Exs. 1–7, 11, 29, 30, 36–44
Day 2: pp. 461–464
Exs. 8–10, 12–19, 27, 28, 31

Average:
Day 1: pp. 461–464
Exs. 1–7, 11, 29, 30, 33, 36–44
Day 2: pp. 461–464
Exs. 8–10, 12, 16–25, 27, 28, 31, 32

Advanced:
Day 1: pp. 461–464
Exs. 1–7, 11, 29, 30, 33, 36–44
Day 2: pp. 461–464
Exs. 9, 10, 17–28*, 31, 32, 34, 35*

Block:
pp. 461–464
Exs. 1–7, 11, 29, 30, 33, 36–44
(with 7.3)
pp. 461–464
Exs. 8–10, 12, 16–25, 27, 28, 31, 32
(with 7.5)

Differentiated Instruction

See *Geometry Best Practices Toolkit* for suggestions on addressing the needs of a diverse classroom.

Homework Check

For a quick check of student understanding of key concepts, go over the following exercises:
Basic: 3, 6, 8, 27, 28
Average: 4, 6, 9, 27, 29
Advanced: 4, 7, 10, 27, 30

Extra Practice

- Student Edition, p. 908
- Chapter 7 Resource Book: Practice levels A, B, C, pp. 49–54

Practice Worksheet

An easily-readable reduced practice page (with answers) for this lesson can be found on p. 430C.

2. The sum of the interior angles of a triangle is 180°; if one angle is 90°, then the other 2 angles must total 90°. Since the triangle is isosceles, these angles must be congruent. Therefore each angle must be half of 90° or 45°.

11.

a	7	11	$5\sqrt{2}$	6	$\sqrt{5}$
b	7	11	$5\sqrt{2}$	6	$\sqrt{5}$
c	$7\sqrt{2}$	$11\sqrt{2}$	10	$6\sqrt{2}$	$\sqrt{10}$

12.

d	5	7	8	$9\sqrt{3}$	$4\sqrt{3}$
e	$5\sqrt{3}$	$7\sqrt{3}$	$8\sqrt{3}$	27	12
f	10	14	16	$18\sqrt{3}$	$8\sqrt{3}$

Reading Strategy

Exercises 4–5 Even though no angle measures are given, students should notice that two sides of each triangle are congruent so they are 45°-45°-90° triangles.

Avoiding Common Errors

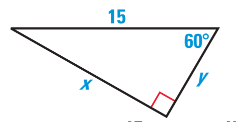
Exercise 10 Some students may be confused because they think that the $\sqrt{3}$ must be on the side of the triangle across from the 60° angle. Show students a triangle where the shorter leg is $2\sqrt{3}$, and have them see that the longer leg is 6 and the hypotenuse is $4\sqrt{3}$. Similarly, show them a 45°-45°-90° triangle where the length of a leg is a multiple of $\sqrt{2}$.

Animated Geometry classzone.com

An **Animated Geometry** activity is available on-line for **Exercises 13–18**. This activity is also available on the **Power Presentations CD-ROM**.

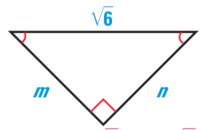
xy ALGEBRA **Find the value of each variable. Write your answers in simplest radical form.**

13.



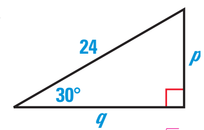
$x = \frac{15}{2}\sqrt{3}$, $y = \frac{15}{2}$

14.



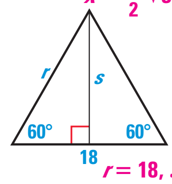
$m = \sqrt{3}$, $n = \sqrt{3}$

15.



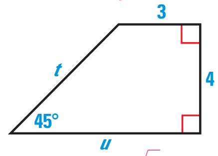
$p = 12$, $q = 12\sqrt{3}$

16.



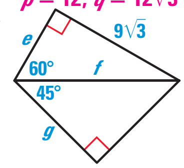
$r = 18$, $s = 9\sqrt{3}$

17.



$t = 4\sqrt{2}$, $u = 7$

18.



$e = 9$, $f = 18$, $g = 9\sqrt{2}$

Animated Geometry at classzone.com

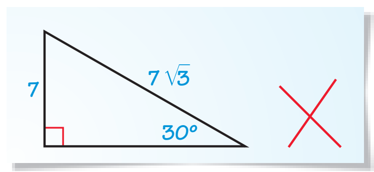
19. ★ **MULTIPLE CHOICE** Which side lengths do *not* represent a 30°-60°-90° triangle? **C**

Ⓐ $\frac{1}{2}, \frac{\sqrt{3}}{2}, 1$ Ⓑ $\sqrt{2}, \sqrt{6}, 2\sqrt{2}$

Ⓒ $\frac{5}{2}, \frac{5\sqrt{3}}{2}, 10$ Ⓓ $3, 3\sqrt{3}, 6$

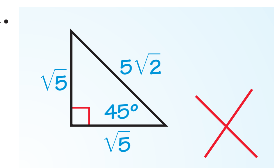
ERROR ANALYSIS ***Describe*** **and correct the error in finding the length of the hypotenuse.**

20.



20. The hypotenuse of a 30°-60°-90° triangle should be $2x$ not $x\sqrt{3}$; if $x = 7$, then the hypotenuse is 14.

21.

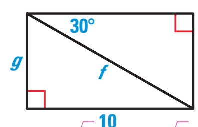


The hypotenuse of a 45°-45°-90° triangle should be $x\sqrt{2}$; if $x = \sqrt{5}$, then the hypotenuse is $\sqrt{10}$.

22. ★ **WRITING** Abigail solved Example 5 on page 459 in a different way. Instead of dividing each side by $\sqrt{3}$, she multiplied each side by $\sqrt{3}$. Does her method work? *Explain* why or why not. **Yes.** ***Sample answer:*** **After she multiplies by $\sqrt{3}$ she would have to divide by 3 to solve for x and find $x = 3\sqrt{3}$.**

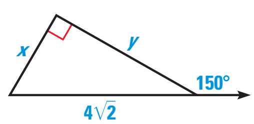
xy ALGEBRA **Find the value of each variable. Write your answers in simplest radical form.**

23.



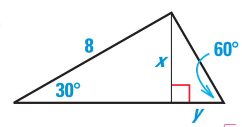
$f = \frac{20\sqrt{3}}{3}$, $g = \frac{10\sqrt{3}}{3}$

24.



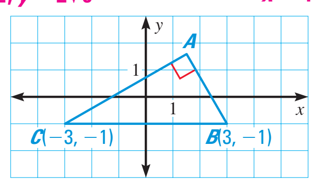
$x = 2\sqrt{2}$, $y = 2\sqrt{6}$

25.



$x = 4$, $y = \frac{4\sqrt{3}}{3}$

C **26.** **CHALLENGE** $\triangle ABC$ is a 30°-60°-90° triangle. Find the coordinates of A. **about (1.5, 1.60)**



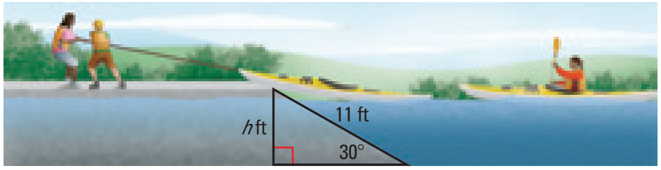
462

PROBLEM SOLVING

EXAMPLE 6 on p. 460 for Ex. 27

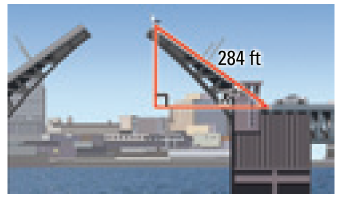
A

27. KAYAK RAMP A ramp is used to launch a kayak. What is the height of an 11 foot ramp when its angle is 30° as shown? **5.5 ft**



@HomeTutor for problem solving help at classzone.com

28. DRAWBRIDGE Each half of the drawbridge is about 284 feet long, as shown. How high does a seagull rise who is on the end of the drawbridge when the angle with measure $x°$ is 30°? 45°? 60°?

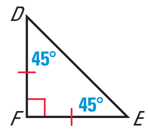


@HomeTutor for problem solving help at classzone.com

142 ft, $142\sqrt{2}$ ft, $142\sqrt{3}$ ft

29. ★ SHORT RESPONSE *Describe* two ways to show that all isosceles right triangles are similar to each other. **See margin.**

30. PROVING THEOREM 7.8 Write a paragraph proof of the 45°-45°-90° Triangle Theorem. **See margin.**



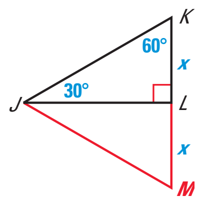
GIVEN ▶ $\triangle DEF$ is a 45°-45°-90° triangle.

PROVE ▶ The hypotenuse is $\sqrt{2}$ times as long as each leg.

B

31. EQUILATERAL TRIANGLE If an equilateral triangle has a side length of 20 inches, find the height of the triangle. **$10\sqrt{3}$ in.**

32. PROVING THEOREM 7.9 Write a paragraph proof of the 30°-60°-90° Triangle Theorem. **See margin.**

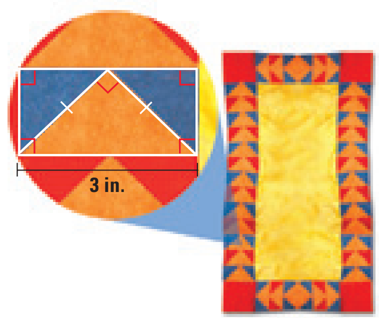


GIVEN ▶ $\triangle JKL$ is a 30°-60°-90° triangle.

PROVE ▶ The hypotenuse is twice as long as the shorter leg and the longer leg is $\sqrt{3}$ times as long as the shorter leg.

Plan for Proof Construct $\triangle JML$ congruent to $\triangle JKL$. Then prove that $\triangle JKM$ is equilateral. Express the lengths of $\overline{JK}$ and $\overline{JL}$ in terms of x.

33. MULTI-STEP PROBLEM You are creating a quilt that will have a traditional "flying geese" border, as shown below.



a. Find all the angle measures of the small blue triangles and the large orange triangles. **45°-45°-90° for all triangles**

b. The width of the border is to be 3 inches. To create the large triangle, you cut a square of fabric in half. Not counting any extra fabric needed for seams, what size square do you need? **$\frac{3\sqrt{2}}{2}$ in. × $\frac{3\sqrt{2}}{2}$ in.**

c. What size square do you need to create each small triangle? **1.5 in. × 1.5 in.**

Avoiding Common Errors

Exercise 32 Some students may write the square of the hypotenuse as $2x^2$, forgetting to write it as $(2x)^2$. Encourage them to start by writing $(\ )^2 + (\ )^2 = (\ )^2$, and then filling in values for the lengths of the three sides.

30. It is given that $\angle D \cong \angle E$, and $\angle F$ is a right angle, so by the Converse of the Base Angles Theorem, $\overline{DF} \cong \overline{EF}$. Then by the Pythagorean Theorem, $DF^2 + EF^2 = DE^2$. By substitution, the equation becomes $DF^2 + DF^2 = DE^2$. By addition, we get $2DF^2 = DE^2$ and a property of square roots allows us to state that $DE = DF \cdot \sqrt{2}$ or by substitution, $DE = EF \cdot \sqrt{2}$.

32. It is given that $\triangle JKL$ is a 30°-60°-90° triangle with x as the side opposite the 30° angle and $\triangle JKL \cong \triangle JML$. Since $m\angle KJL$ is 30° and $m\angle MJL$ is also 30°, angle addition shows that $\angle KJM$ measures 60°. In addition, the definition of an equiangular triangle shows that $\triangle JKM$ is equiangular and since $\triangle JKM$ is equiangular, it is also equilateral. This allows us to state that since $KM = 2x$, then $JK = 2x$. Therefore, JK which is the hypotenuse of $\triangle JKL$ is twice as long as the shorter leg of this triangle. Using $\triangle JKL$, the Pythagorean Theorem states that $JL^2 + LK^2 = JK^2$. The Substitution Property of Equality allows us to rewrite this equation as $JL^2 + x^2 = (2x)^2$. A property of exponents simplifies the equation to $JL^2 + x^2 = 4x^2$, and subtraction simplifies the equation to $JL^2 = 3x^2$. Finally, a property of square roots simplifies the equation to $JL = x\sqrt{3}$.

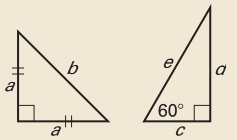
29. *Sample answer:* Method 1. Use the Angle-Angle Similarity Postulate, because by definition of an isosceles triangle, the base angles must be the same and in a right isosceles triangle, the angles are 45°. Method 2. Use the Side-Angle-Side Similarity Theorem, because a right angle is always congruent to another right angle and the ratio of the lengths of the corresponding sides of two isosceles right triangles will always be the same.

5 ASSESS AND RETEACH

Daily Homework Quiz

Transparency Available

Use these triangles for Exercises 1–4.



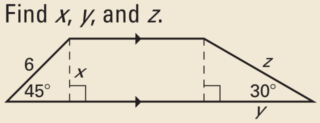
1. Find a if $b = 10\sqrt{2}$. **10**

2. Find b if $a = 19$. **$19\sqrt{2}$**

3. Find d and e if $c = 4$.
$d = 4\sqrt{3}$, $e = 8$

4. Find c and d if $e = 50\sqrt{3}$.
$c = 25\sqrt{3}$, $d = 75$

5. Find x, y, and z.



$x = 3\sqrt{2}$, $y = 3\sqrt{6}$, $z = 6\sqrt{2}$

Online Quiz

Available at **classzone.com**

Diagnosis/Remediation

- Practice A, B, C in Chapter 7 Resource Book, pp. 49–54
- Study Guide in Chapter 7 Resource Book, pp. 55–56
- Practice Workbook, pp. 133–135
- @HomeTutor

Challenge

Additional challenge is available in the Chapter 7 Resource Book, p. 60.

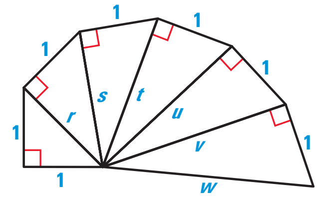
Quiz

An easily-readable reduced copy of the quiz (with answers) on Lessons 7.3–7.4 from the Assessment Book can be found on p. 430G.

34a, 35a. See Additional Answers beginning on p. AA1.

34b. The left most triangle with sides of 1; the triangle must be a 45°–45°–90° because it is an isosceles right triangle.

C **34.** ★ **EXTENDED RESPONSE** Use the figure at the right. You can use the fact that the converses of the 45°-45°-90° Triangle Theorem and the 30°-60°-90° Triangle Theorem are true.



a. Find the values of r, s, t, u, v, and w. *Explain* the procedure you used to find the values. **See margin.**

b. Which of the triangles, if any, is a 45°-45°-90° triangle? *Explain.*

c. Which of the triangles, if any, is a 30°-60°-90° triangle? *Explain.*
The triangle with t as the hypotenuse; the side lengths fit those given in Theorem 7.9.

35. **CHALLENGE** In quadrilateral $QRST$, $m\angle R = 60°$, $m\angle T = 90°$, $QR = RS$, $ST = 8$, $TQ = 8$, and $\overline{RT}$ and $\overline{QS}$ intersect at point Z.

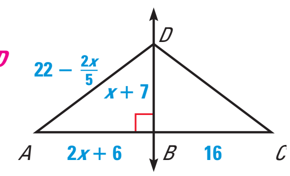
a. Draw a diagram. **See margin.**

b. *Explain* why $\triangle RQT \cong \triangle RST$. **Side-Side-Side Congruence Postulate**

c. Which is longer, QS or RT? *Explain.* **RT; $QS = 8\sqrt{2}$ and $RT = 4\sqrt{6} + 4\sqrt{2}$.**

MIXED REVIEW

In the diagram, $\overleftrightarrow{BD}$ is the perpendicular bisector of $\overline{AC}$. *(p. 303)*



36. Which pairs of segment lengths are equal? **$AB = BC$, $AD = CD$**

37. What is the value of x? **5**

38. Find CD. **20**

Is it possible to build a triangle using the given side lengths? *(p. 328)*

39. 4, 4, and 7 **yes**

40. 3, 3, and $9\sqrt{2}$ **no**

41. 7, 15, and 21 **yes**

PREVIEW
Prepare for Lesson 7.5 in Exs. 42–44.

Tell whether the given side lengths form a right triangle. *(p. 441)*

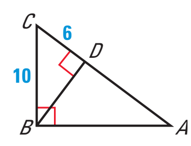
42. 21, 22, and $5\sqrt{37}$ **right triangle**

43. $\frac{3}{2}$, 2, and $\frac{5}{2}$ **right triangle**

44. 8, 10, and 14 **not a right triangle**

QUIZ for Lessons 7.3–7.4

In Exercises 1 and 2, use the diagram. *(p. 449)*

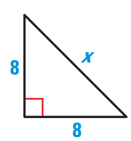


1. Which segment's length is the geometric mean of AC and CD? **CB**

2. Find BD, AD, and AB. **8, $\frac{32}{3}$, $\frac{40}{3}$**

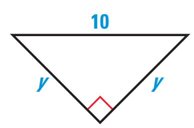
Find the values of the variable(s). Write your answer(s) in simplest radical form. *(p. 457)*

3.



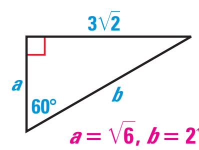
$8\sqrt{2}$

4.



$5\sqrt{2}$

5.



$a = \sqrt{6}$, $b = 2\sqrt{6}$